Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #16

Integration by Parts

$$\int x \cos x \, dx$$

Suppose that u and v are functions of x and we differentiate the product uv.

$$\frac{duv}{dx} = v\frac{du}{dx} + u\,\frac{dv}{dx}$$

Suppose we start with this derivative and work toward an anti-derivative.

The anti-derivative of the left side is easy: uv

On the right we get

$$\int v\,u'\,dx + \int u\,v'dx$$

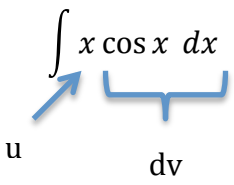
We usually write $du = u'dx$ and $dv = v'dx$.

So

$$uv = \int v\,du + \int u\,dv$$

$$\int u\,dv = uv - \int v\,du$$

We take our integral and break it into two parts.



So $u = x$ and $dv = \cos x \, dx$

Then also $du = dx$ and $v = \sin x$

Integration by parts says:

$$\int x \cos x \, dx = uv - \int v \, du$$

$$= x \sin x - \int \sin x \, dx$$
$$= x \sin x + \cos x + C$$

Check:

$$(x \sin x + \cos x)' = \sin x + x \cos x - \sin x = x \cos x$$

$$\int u \, dv = uv - \int v \, du$$